Report

A Visual Sense of Number

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Summary

Evidence exists for a nonverbal capacity for the apprehension of number, in humans [1] (including infants [2, 3]) and in other primates [4–6]. Here, we show that perceived numerosity is susceptible to adaptation, like primary visual properties of a scene, such as color, contrast, size, and speed. Apparent numerosity was decreased by adaptation to large numbers of dots and increased by adaptation to small numbers, the effect depending entirely on the numerosity of the adaptor, not on contrast, size, orientation, or pixel density, and occurring with very low adaptor contrasts. We suggest that the visual system has the capacity to estimate numerosity and that it is an independent primary visual property, not reducible to others like spatial frequency or density of texture [7].

Results and Discussion

Jevons, a 19th century economist, rather than counting beans, assessed his accuracy in estimating the number of beans in a box at a single glance [8]. He made no errors at four or below but became increasingly inaccurate as the number of beans increased beyond four. Subsequent studies have confirmed his findings and the lack of errors below five has led to the concept of *subitizing* [9–12], usually presumed to be a separate process allowing rapid apprehension of the numerosity of collections containing fewer than five objects. The perception of larger numbers is usually assumed to require slower and more cognitive processes, like counting.

All primary visual properties are susceptible to *adaptation*, sometimes giving rise to dramatic aftereffects, like the waterfall illusion [13], and changes in color, size, distance, spatial frequency, and orientation. If numerosity was a primary visual property, like color or motion, it too should be prone to adaptation. The online demonstration shows that it is. After 30 s adaptation to the two different adaptor patches, the two subsequent patches appear to differ considerably in numerosity (whereas inspection after adaptation wears off, or counting, shows that they both number 30 dots). We quantified adaptation effects by asking subjects whether a *test* stimulus (of variable numerosity), presented to the region that had been adapted, appeared more or less numerous than a *probe* stimulus (of fixed

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numerosity), presented to a different unadapted position a little later. The proportion of trials where the test appeared more numerous than the probe was plotted against test numerosity and fitted with cumulative Gaussian functions whose mean estimates the point of subjective equality (PSE) between test and probe, and standard deviation the threshold for discriminating between the two (the just-noticeable difference [jnd]). Figure 1A shows sample psychometric functions for a 30 element probe, with and without adaptation to a 400 element stimulus. The ratio of the matched test to probe increases from unity (30 dots) with no adaptation to more than 3 (100 dots) after adaptation (we increased the test number to compensate for the reduction in its apparent numerosity). Note also that that after adaptation the psychometric function is steeper (on logarithmic coordinates), implying a smaller jnd.

We first measured the effect of adapting to a large number (400) of dots as a function of number of dots in the probe (Figure 1B). The amount of adaptation was fairly constant with probe numerosity down to about 12 dots and then decreased as the probe approached the subitizing range. The precision of the match, given by the jnd or Weber fraction (jnd expressed as a fraction of dot number), did not deteriorate during adaptation, the average percentage Weber fractions for unadapted and adapted conditions being 28% for unadapted and 26% for the adapted conditions.

We next investigated whether adaptation to small numbers can cause an increase in apparent numerosity. The red circles of Figure 2 show that adaptation occurred in both directions: Adaptation to small numbers increased apparent numerosity (so the matched number decreased), and adaptation to large numbers decreased apparent numerosity. Adaptation to 50 dots (the number of the probe) had no effect, with the amount of adaptation increasing with the difference between adapt and probe number. The curves of both subjects were well fit by linear regression on log coordinates, with a slope around 0.25.

In order to test whether adaptation depends on numerosity per se or is derived from other factors, like texture density [7], we performed a number of controls. We first varied the size of the adaptor and test dots, in order to vary pixel density. In the above-described study (red circles of Figure 2), both adaptor and test dots were circles of 6 pixel (20 arcmin) diameter (28 pixel area). We repeated the experiment with square adaptor stimuli of 8 × 8 pixels (64 pixels) and test stimuli of 3 × 3 pixels (9 pixels, 1/7 as many as the adaptor). If pixel density were the relevant attribute, the curves of Figure 2 should shift leftwards by a factor of 7, so the null point occurs when adaptor and test pixel density are matched (for adaptation dot number of 7). This clearly does not occur. For naive observer PB, the curves remain superimposed; for DB, there is a slight shift in the opposite direction.

We also examined the effect of adaptor contrast. As Figure 2C shows, contrast of adaptor dots had little effect on the magnitude of adaptation. At contrasts as low as 12%, the adaptation effect is still nearly 2-fold, dropping only near detection threshold. It appears that the only factor that affects adaptation is numerosity, not density, orientation, or contrast.
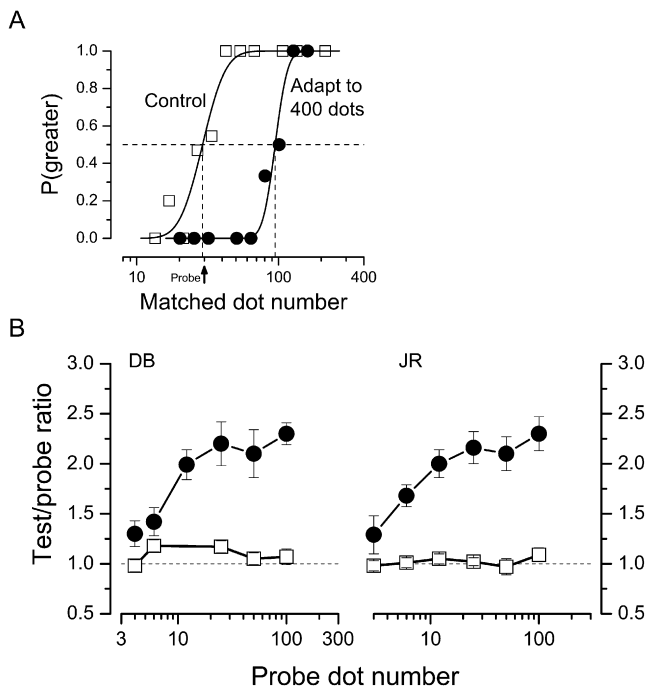
Figure 1. The Effect of Adaptation on Numerosity

(A) Sample psychometric functions with (filled circles) and without (open squares) adaptation, plotting the proportion of trials where the probe seemed more numerous, as a function of number of test dots. The vertical dashed lines indicate the PSE of the match, about three times higher than the probe number (indicated by the arrow) after adaptation.

(B) Magnitude of adaptation (ratio of test to probe dot number at PSE) as a function of the number of dots in the probe (symbols as for Figure 1A). The error bars (here and elsewhere) show ± 1 standard error of the mean (SEM), calculated by bootstrap. For a wide range of numerosities, adaptation caused a doubling of the matched number.

As a direct control for the effects of texture, we next adapted to vertical elements and tested either vertical or horizontal elements. As the bar graphs of Figure 3A show, there was little difference in the magnitude of the effects. If texture or spatial frequency was being adapted, one would expect some specificity for orientation. We also performed discriminations (without adaptation) for patterns of completely different pixel density, orientation, Fourier transform, etc. An example is shown in the psychometric functions of Figure 3B, where the test or probe could be either small 5 × 5 pixel (16.5 × 16.5 arcmin) squares or 20 × 5 pixel rectangles, randomly vertical or horizontal. Neither the PSE nor the width of the curves depended on the type of stimuli being compared, even though the stimuli were visually completely different, varied by a factor of 4 in pixel density and Michelson contrast, and had completely different Fourier power spectra. All these results agree with a recent study (Jun Zhang, personal communication) showing that apparent numerosity of a field of dots can be reduced by the addition of links between some dots: The linked pair contributes to the numerosity as a single entity, rather as two separate dots.

We propose that just as we have a direct visual sense of the *reddishness* of half a dozen ripe cherries, so we do of their *six-ishness*. In other words there are distinct qualia [14] for numerosity, as there are for color, brightness, and contrast. One of the more fascinating aspects of this study—as readers can verify for themselves with the online demonstration—is that
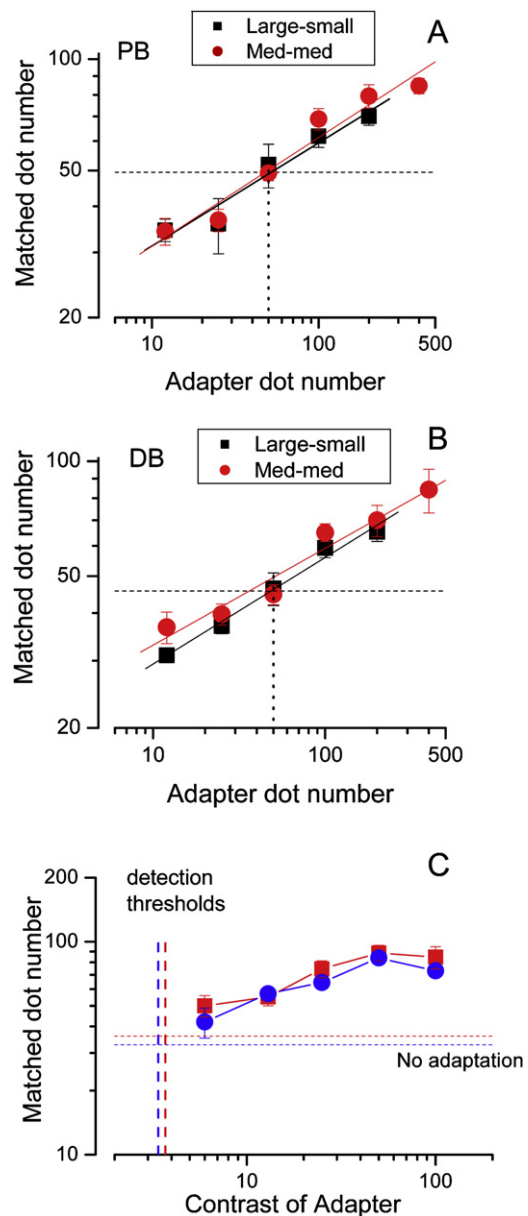

Figure 2. Effect of Numerosity and Contrast of the Adaptor

(A and B) Effect of adaptor numerosity and density on apparent numerosity of a 50 dot probe. The red circles refer to adaptor and test dots of 6 pixel (20 arcmin) diameter and the squares to adapters of 8 × 8 and tests of 3 × 3 pixels (7 times more adapt than test pixels for matched numerosity). In all cases, the adapters were of 50% Michelson contrast, and the tests were 100%. Adaptation occurs for both high and low adaptation numbers and is independent of pixel density.

(C) Effect of adaptor contrast on apparent numerosity of a 30 dot probe (red symbols indicate DB, blue symbols indicate PB). The vertical dashed lines indicate the contrast threshold for detection of the patterns (see the Experimental Procedures), and the horizontal lines indicate the matches with no adaptation. Adaptation effects were pronounced down to near-threshold contrasts.

although the total apparent number of dots is greatly reduced after adaptation, no particular dots seem to be missing. This reinforces old and more recent evidence [15–17] suggesting that the perceived richness of our perceptual world is very much an illusion. Although we seem to perceive 30 or 50 or 100 individual dots occupying very specific positions, this
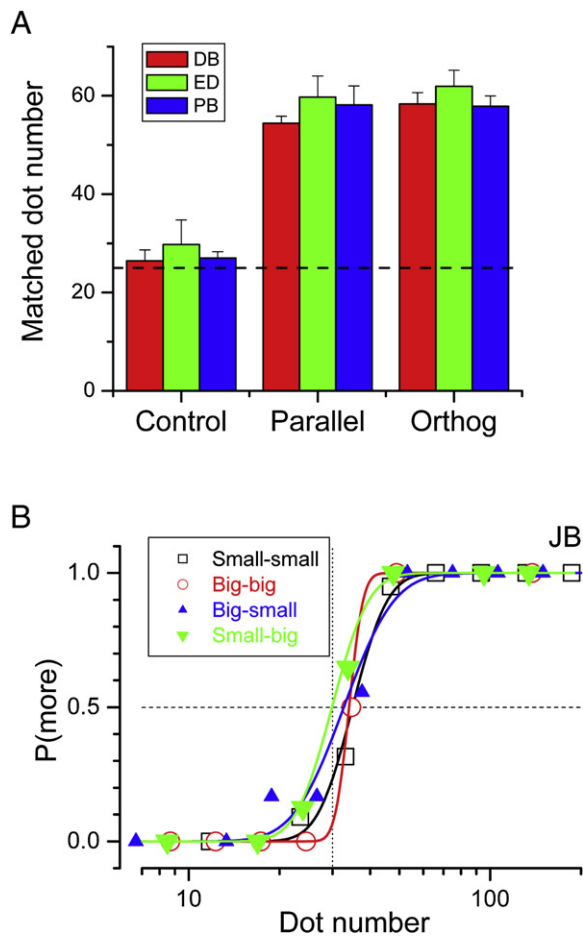
A

B

Figure 3. Effect of Element Size and Shape

(A) Effect of adaptor orientation. Subjects adapted to a field of 200 vertical elements (3 × 10 pixels) and matched a field of either vertical or horizontal same-sized elements to a probe (same orientation as test). The effects of orthogonal and parallel adapters were similar.

(B) Psychometric curves for matching numerosity of element arrays that were the same (5 × 5 or 5 × 20 pixels), small with large, or large with small. Element size and shape has very little effect of either PSE or Weber fraction (given by the function width), suggesting that the matches were based solely on number of elements.

cannot be the case because adaptation could not reduce or increase the total number of dots without annihilating or creating them. Rather, it would seem that what we see derives from a statistical description of the scene, where some aspects of the elements (color, shape, contrast, etc.) are encoded, together with a rough (±30%) estimate of their numerosity.

Recent studies have demonstrated the existence of neurons broadly tuned for number in the parietal cortex of macaque monkeys [4, 18, 19]. Functional magnetic resonance imaging (fMRI) studies also point to their existence in a the intraparietal sulcus in humans, both for symbolic [12, 20, 21] and nonsymbolic [22] representation of numbers. These neurones are likely candidates for the physiological substrate of the visual sense of number, and, like most neurones, they are probably adaptable. Vision has formidable built-in computational powers, correcting for variation in image size with distance, in image shape with tilt, and in image spectral composition with changes in illuminant, allowing for approximately constant perception of object size, shape, and color; it can also segment images,

a difficult computational task [23]. It should come as no surprise that it can provide approximate estimates of number.

Experimental Procedures

Stimuli
Stimuli were generated by a framestore (Cambridge Research Systems VSG Visage) and displayed on the face of a Hitachi Accuvue monitor at 170 Hz framerate, with a resolution of 640 × 480 pixels and mean luminance of 18 cd/m². The 37 × 28 cm screen subtended 35° × 26.5° at the viewing distance of 60 cm (each pixel 3.3 arcmin wide). The stimuli were fields of small disks (of 6° diameter, unless otherwise stated), randomly positioned within a circle of 10° diameter (similar in appearance to those of the demo in the Supplemental Data available online). The disks were half bright and half dark, of 100% contrast (unless otherwise stated).

Procedure
Subjects fixated on a fixation spot at the center of the screen. The adaptation stimuli were centered 7° away from fixation, above left for half the sessions and below right for the others. The test stimulus was displayed in the same position as the adaptor for 600 ms, and then the probe stimulus was displayed for 600 ms, directly above or below the test stimulus (all stimuli were separated by a pause of 400 ms). Subjects adapted for 30 s at the beginning of each session, with 7 s top-up adaptation between trials. On each trial, subjects were required to report whether the probe appeared more or less numerous than the test, guessing if unsure. After each trial, an adaptive algorithm (QUEST [24]) estimated the PSE, which, after addition of a random quantity (drawn from a log Gaussian distribution of standard deviation 0.15 log units) determined the probe number for the following trial. The technique ensured an approximately equal number of right and left button presses, as well as placing most trials at a numerosity for the estimation of the best PSE and curve slope. The proportion of "greater" trials was plotted against the logarithm of probe numerosity and fit with a cumulative Gaussian function (see Figure 1A), whose mean yielded an estimate of PSE and standard deviation an estimate of jnd.

Contrast thresholds (reported in Figure 2C) were measured by a two-alternative forced-choice procedure. Half of the dots (above or below a diagonal line radiating from fixation) were removed, and subjects were required to identify in which half the dots were confined. Again the QUEST [24] algorithm homed in near threshold, and threshold was calculated by Gaussian fit (allowing for guessing).

Subjects
Four subjects were measured systematically for most conditions—the two authors and two others naive to the goals of the study (PB and ED). Sample results are shown in the figures.

Supplemental Data

A demonstration of adaptation to numerosity is available at http://www.current-biology.com/cgi/content/full/18/6/425/DC1/.

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